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# Computation of Shortest Path in a Fuzzy Network Using Triangular Intuitionistic Fuzzy Number

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Abstract— A new method is proposed for finding the Shortest Path Problem (SPP) with Triangular Intuitionistic Fuzzy numbers (TIFN), based on Bellman Dynamic Programming algorithm. Here, we compute the shortest path from a specified vertex to all other vertices in a fuzzy network and a numerical example is also provided.

Index Terms— TIFN, SPP, Bellman dynamic programming.

## 1 Introduction

The SPP requires determining the shortest route between a source and a destination in a transportation network. In some applications, the numbers associated with the edges of networks may represent characteristics other than lengths, and we may want the optimum paths, where optimum can be defined by different criteria. The SPP is one of the most fundamental and well-known combinatorial optimization problems that appear in many applications as a sub-problem. The length of arcs in the network represents travelling time, cost, distance or other variables. In 1980 Dubois and Prade [4] first introduce fuzzy SPP. Okada and Soper [5] developed an algorithm based on multiple labelling approach which is useful to generate number of non-dominated paths. Applying minimum concept they have introduced an order relation between fuzzy numbers. Applying extension principle Klein [6] , has given an algorithm which results in a dominated path on a acyclic network. This paper is organized as follows. In section 2, preliminary concepts and definitions are given. The procedure for finding SPP using TIFN is developed in section 3.An illustrative example is provided in section 4 to find the shortest path. The last section draws some concluding remarks.

## 2 PREREQUISITES

### 2.1 Intuitionistic Fuzzy Number (IFN):

Let $A=\{x,\mu_A(x),\gamma_A(x)/x\in X\}$ be an IFS, then we call $(\mu_A(x),\gamma_A(x))$ an IFN.We denote it by $(\langle a,b,c\rangle,\langle e,f,g\rangle)$ where $\langle a,b,c\rangle$ and $\langle e,f,g\rangle\in F(I)$, $I=[0,1]$, $0\le c+g\le 1$.

### 2.2 Intuitionistic Fuzzy Set (IFS):

An IFS A in X is given by $A=\{x,\mu_A(x),\gamma_A(x)/x\in X\}$ where $\mu_A(x)$ : $X\to[0,1]$ and $\gamma_A(X)$: $X\to[0,1]$ and for every $x\in X, 0\le\mu_A(x)+\gamma_A(x)\le 1$.

### 2.3 Triangular Intuitionistic Fuzzy Number (TIFN) and its arithmetic:

A TIFN 'A' is given by $A=(\langle x,y,z\rangle,\langle l,m,n\rangle)$ with $\langle l,m,n\rangle\le\langle x,y,z\rangle^c$ i.e., either $l\ge y$, $m\ge z$, (or) $m\le x$, $n\le y$ are membership and non- membership fuzzy numbers of A.

The additions of two TIFN are as follows.

For two triangular intuitionistic fuzzy numbers $A=(\langle a_1,b_1,c_1\rangle:\mu_A,\langle e_1,f_1,g_1\rangle:\gamma_A)$ and $B=(\langle a_2,b_2,c_2\rangle:\mu_B,\langle e_2,f_2,g_2\rangle:\gamma_B)$ with $\mu_A\ne\mu_B$ and $\gamma_A\ne\gamma_B$, define A+B=

$$\begin{pmatrix}\langle a_1+a_2,b_1+b_2,c_1+c_2\rangle:Min(\mu_A,\mu_B),\\ \langle e_1+e_2,f_1+f_2,g_1+g_2\rangle:Max(\gamma_A,\gamma_B)\end{pmatrix}$$

### 2.4 Signed Distance:

For each $\tilde{d}=(\langle a,b,c\rangle,\langle e,f,g\rangle)\in F$, the signed distance of $\tilde{d}$ measured from 0 is defined by $d(\tilde{d},0)=¼[(a+e),2(b+f),(c+g)]$. From this definition for each

$$\tilde{d}=\begin{bmatrix}(\langle a_1,a_2,a_3\rangle,\langle a_4,a_5,a_6\rangle),(\langle b_1,b_2,b_3\rangle,\langle b_4,b_5,b_6\rangle),\\ (\langle c_1,c_2,c_3\rangle,\langle c_4,c_5,c_6\rangle)\end{bmatrix},$$

we obtain

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$$P^0_{ij} = \frac{1}{4}\left[\begin{array}{l}(a_1 + a_4, a_2 + a_5, a_3 + a_6) + 2(b_1 + b_4, b_2 + b_5, b_3 + b_6),\\ (c_1 + c_4, c_2 + c_5, c_3 + c_6)\end{array}\right]$$

## 3. Shortest Path Length Procedure in Intuitionistic sense:

According to Bellman's equation, a Dynamic Programming formulation for the SPP can be given as follows:

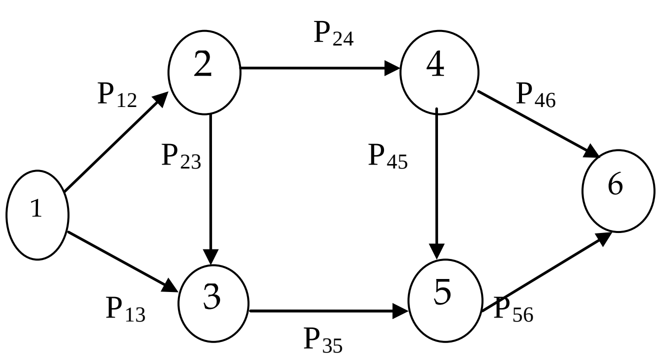
Given a network with an acyclic directed graph G = (V, E) with n vertices numbered from 1 to n such that 1 is the source and n is the destination. Then we have

$f(n) = 0$

$f(i) = \min_{i<j}\{p_{ij} + f(j) / \langle i, j\rangle \in E\}$ ---------------- (1)

Here $p_{ij}$ is the weight of the directed edge $\langle i, j\rangle$ and f(i) is the length of the shortest path from vertex i to vertex n. From the following figure, the solution of Dynamic Programming can be derived as follows:

## 4. Illustrative Example




Consider a network with the triangular intuitionistic fuzzy arc lengths as shown below. The arc lengths are assumed to be

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$P_{12} = (\langle 15,27,38\rangle, \langle 35,47,52\rangle)$
$P_{13} = (\langle 23,32,45\rangle, \langle 35,48,52\rangle)$ ;
$P_{23} = (\langle 31,46,52\rangle, \langle 50,58,65\rangle)$
$P_{24} = (\langle 31,38,49\rangle, \langle 40,52,60\rangle)$;
$P_{35} = (\langle 12,18,25\rangle, \langle 22,28,35\rangle)$;
$P_{45} = (\langle 12,35,48\rangle, \langle 42,49,50\rangle)$
$P_{46} = (\langle 18,25,32\rangle, \langle 28,37,45\rangle)$;
$P_{56} = (\langle 21,32,48\rangle, \langle 41,52,61\rangle)$

The possible paths are as follows:

$P_1$ : 1-2-4-5-6 ;
$L_1 = (\langle 79,132,183\rangle, \langle 158,200,223\rangle)$

$P_2$:1-2-4-6;
$L_2 = (\langle 64,90,119\rangle, \langle 103,136,157\rangle)$

$P_3$:1-2-3-5-6;
$L_3 = (\langle 79,123,163\rangle, \langle 148,185,213\rangle)$

$P_4$:1-3-5-6;
$L_4 = (\langle 56,82,118\rangle, \langle 98,128,148\rangle)$

The solution of Dynamic Programming can be derived as follows

$f(6) = 0,\ f(5) = P_{56} = (\langle 21,32,48\rangle, \langle 41,52,61\rangle)$

$f(4) = \min_{4<j}\{p_{4j} + f(j) / \langle 4, j\rangle \in E\}$
$= \min\{P_{46}, P_{45} + P_{56}\}$
$= P_{46} = (\langle 18,25,32\rangle, \langle 28,37,45\rangle)$

$f(3) = \min_{3<j}\{p_{3j} + f(j) / \langle 3, j\rangle \in E\}$
$= P_{35} + f(5) = P_{35} + P_{56}$
$= (\langle 33,50,73\rangle, \langle 63,80,96\rangle)$

Similarly, we obtain

$f(2) = \min\{P_{24}+f(4),\ P_{24} + P_{25}+f(5)\}$
$= P_{24} + P_{46}$
$= (\langle 49,63,81\rangle, \langle 68,89,105\rangle)$

$f(1) = \min\{P_{12}+f(2), P_{13}+f(3), P_{12}+P_{23}+f(3),$
$P_{12}+P_{24}+f(4),\ P_{12}+P_{24}+P_{45}+f(5)\}$

$f(1) = (\langle 56,82,118\rangle, \langle 98,128,148\rangle)$

**To compute the shortest path**

In this problem we consider is that the edge weight in the network denoted by $p_{ij}$ and the edge weight should be expressed using fuzzy linguistics, and also this used in TIFN.

$\tilde{p}_{ij} = (p_{ij} - \Delta_{ij1}, p_{ij}, p_{ij} + \Delta_{ij2})$

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.................................... (2)

Where $0 < \Delta_{ij} < p_{ij}$, $\Delta_{ij} > 0$ since $\Delta_{ij1}$ and $\Delta_{ij2}$ should be determined by the Decision Maker. Now consider the fuzzy case. We look for inequalities that satisfy

$$p_{ii_1} + p_{i_1 i_2} + \ldots\ldots + p_{i_{m(i)^n}} \le p_{ij_1} + p_{j_1 j_2} + \ldots\ldots + p_{j_{m(i)^n}},$$
$$\forall i < j, \langle i, j \rangle \in E$$

When i = 1, $p_{13} + f(3) < p_{12} + f(2)$

i.e, $p_{13} + p_{35} + p_{56} < p_{12} + p_{23} + f(3)$

$< p_{12} + p_{24} + p_{35} + p_{56}$

( Or ) $p_{13} + f(3) < p_{12} + p_{24} + p_{45} + p_{56}$

When i = 2, $p_{24} + f(4) < p_{23} + f(3)$

$p_{24} + p_{46} < p_{23} + p_{35} + p_{56}$

When i = 4, $p_{46} + f(6) < p_{45} + f(5)$

$P_{46} < p_{45} + p_{56}$

Then the parameters of

$$\Delta_{ii_1} + \Delta_{i_1 i_2} + \ldots\ldots + \Delta_{i_{m(i)^n}} \le \Delta_{ij_1} + \Delta_{j_1 j_2} + + \ldots\ldots + \Delta_{j_{m(i)^n}}, \forall i$$

i< j, $\langle i, j \rangle \in E$ based on the above inequalities are derived as

$\Delta_{13} + \Delta_{35} + \Delta_{56} < \Delta_{12} + \Delta_{24} + \Delta_{46},$

$< \Delta_{12} + \Delta_{24} + \Delta_{45} + \Delta_{56},$

............................ (3)

$\Delta_{24} + \Delta_{46} < \Delta_{23} + \Delta_{35} + \Delta_{56},$

$\Delta_{46} < \Delta_{45} + \Delta_{56}$

If the Decision Maker chooses the values of parameters:

$\Delta_{12} = (\langle 2,3,5\rangle, \langle 4,6,7\rangle)$; $\Delta_{13} = (\langle 4,5,6\rangle, \langle 7,8,9\rangle)$;

$\Delta_{23} = (\langle 3,7,9\rangle, \langle 8,10,11\rangle)$; $\Delta_{24} = (\langle 2,5,6\rangle, \langle 6,8,9\rangle)$;

$\Delta_{35} = (\langle 1,3,5\rangle, \langle 4,6,7\rangle)$; $\Delta_{45} = (\langle 3,5,6\rangle, \langle 6,8,10\rangle)$;

$\Delta_{46} = (\langle 5,7,8\rangle, \langle 9,10,12\rangle)$; $\Delta_{56} = (\langle 6,7,9\rangle, \langle 8,10,11\rangle)$ to satisfy the conditions in (3), then the fuzzy numbers can be determined as

$$\tilde{p}_{12} = \begin{bmatrix} (\langle 12,23,32\rangle, \langle 29,40,44\rangle), (\langle 15,27,38\rangle, \langle 35,47,52\rangle), \\ (\langle 20,34,49\rangle, \langle 45,60,67\rangle) \end{bmatrix}$$

$$\tilde{p}_{13} = \begin{bmatrix} (\langle 19,27,39\rangle, \langle 28,40,43\rangle), (\langle 23,32,45\rangle, \langle 35,48,52\rangle), \\ (\langle 31,42,58\rangle, \langle 49,64,70\rangle) \end{bmatrix}$$

$$\tilde{p}_{23} = \begin{bmatrix} (\langle 28,41,45\rangle, \langle 44,50,56\rangle), (\langle 31,46,52\rangle, \langle 50,58,65\rangle), \\ (\langle 37,58,68\rangle, \langle 64,76,85\rangle) \end{bmatrix}$$

$$\tilde{p}_{24} = \begin{bmatrix} (\langle 27,33,43\rangle, \langle 33,44,51\rangle), (\langle 31,38,49\rangle, \langle 40,52,60\rangle), \\ (\langle 37,48,61\rangle, \langle 53,68,78\rangle) \end{bmatrix}$$

$$\tilde{p}_{35} = \begin{bmatrix} (\langle 8,13,18\rangle, \langle 16,20,26\rangle), (\langle 12,18,25\rangle, \langle 22,28,35\rangle), \\ (\langle 17,26,37\rangle, \langle 32,42,51\rangle) \end{bmatrix}$$

$$\tilde{p}_{45} = \begin{bmatrix} (\langle 5,27,39\rangle, \langle 32,37,37\rangle), (\langle 12,35,48\rangle, \langle 42,49,50\rangle), \\ (\langle 22,48,63\rangle, \langle 58,69,73\rangle) \end{bmatrix}$$

$$\tilde{p}_{46} = \begin{bmatrix} (\langle 14,20,24\rangle, \langle 22,28,35\rangle), (\langle 18,25,32\rangle, \langle 28,37,45\rangle), \\ (\langle 27,37,48\rangle, \langle 43,56,67\rangle) \end{bmatrix}$$

$$\tilde{p}_{56} = \begin{bmatrix} (\langle 16,26,41\rangle, \langle 34,44,52\rangle), (\langle 21,32,48\rangle, \langle 41,52,61\rangle), \\ (\langle 33,45,64\rangle, \langle 56,70,81\rangle) \end{bmatrix}$$

From (2.4), we obtain the following estimate of the edge weights in the fuzzy sense:

$P^0_{12} = \langle 51.5, 71.25, 93\rangle$ ; $p^0_{13} = \langle 60.75, 83.25, 101\rangle$ ; $p^0_{23} = \langle 83.75, 108.25, 122\rangle$; $p^0_{24} = \langle 73, 93.25, 112.75\rangle$ ; $p^0_{35} = \langle 35.25, 48.25, 63\rangle$ ; $p^0_{45} = \langle 56.25, 87.25, 102\rangle$; $p^0_{46} = \langle 49.5, 66.25, 82\rangle$ ; $p^0_{56} = \langle 65.75, 88.25, 114\rangle$ ;

The fuzzy network G = (V, E) with $\{p^0_{ij} / \langle i, j\rangle \in E\}$.

Where $f^0(1) = p^0_{12} + f(3)$

$= p^0_{12} + p^0_{35} + f(5)$

$= p^0_{12} + p^0_{35} + p^0_{56},$

The fuzzy shortest path is 1-3-5-6 with length $\langle 161.75, 219.75, 278\rangle$.

## 5 Conclusion

In this paper, a shortest path is obtained using a procedure based on Bellman Dynamic Programming technique in a fuzzy network. The arc lengths are considered as uncertain and are characterized by triangular intuitionistic fuzzy numbers. The efficiency of the method is tested by means of an example.

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